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Further results on base sequences, disjoint complementary sequences, OD(4t; t, t, t, t) and the excess of Hadamard matrices

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Further results on base sequences, disjoint complementary sequences, OD($4t; t, t, t, t$) and the excess of Hadamard matrices

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Abstract. We obtain new base sequences, that is four sequences of lengths $m + p$, $m + p$, m, m, with p odd, which have zero auto correlation function which can be used with Yang numbers and four disjoint complementary sequences (and matrices) with zero non-periodic (periodic) auto correlation function to form longer sequences.

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The Hadamard matrix with greatest known excess for these new t is then listed.

1. Definitions and Introduction

We follow closely the work of Cohen [1], and Jenkins [4] and refer the reader to those papers for most definitions and constructions.

Four sequences of elements $+1, -1$ of lengths $m + p$, $m + p$, m, m, with p odd, which have zero non-periodic auto correlation function are called *base sequences*. In [1] base sequences are displayed for lengths $m + 1$, $m + 1$, m, m, for $m + 1 \in \{2, 3, \ldots, 18, 21, 24, 30\}$.

In Table 1 we give new base sequences for m =12, 13, 15, 16, 17, 18, 25 and $p = 1$. Some of these are new in the sense that they correspond to new decompositions into squares of $2m + 1$.

If X and Y are Golay sequences of length m, $\{1, X\}, \{1, -X\}, \{Y\}, \{Y\}$ are base sequences of lengths $m + 1$, $m + 1$, m, m. So base sequences exist for all $m = 2^a 10^b 26^c$, a, b, c non-negative integers, $p = 1$ and $m \in \{1, \ldots, 18, 20, 23, 25, 29\}$.

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Base sequences are crucial to Yang's [10], [11], [12] constructions for finding longer T-sequences of odd length.

Four $(1,-1)$ sequences $A = (X,\ U,\ Y,\ V)$ where

$$X = \{x_1 = 1,\ x_2,\ x_3, \ldots,\ x_m,\ -x_m, \ldots,\ -x_3,\ -x_2,\ -x_1 = -1\},$$
$$U = \{u_1 = 1,\ u_2,\ u_3, \ldots,\ u_m,\ -u_m, \ldots,\ -u_3,\ -u_2,\ 1\},$$
$$Y = \{y_1,\ y_2, \ldots,\ y_{m-1},\ y_m,\ y_{m-1}, \ldots,\ y_2,\ y_1\},$$
$$V = \{v_1,\ v_2, \ldots,\ v_{m-1},\ v_m,\ v_{m-1}, \ldots,\ v_2,\ v_1\},$$

which have zero non periodic auto correlation function and $8m - 6$ the sum of two squares, or four $(1,-1)$ sequences $A = (X,\ U,\ Y,\ V)$, where

$$X = \{x_1 = 1,\ x_2,\ x_3, \ldots,\ x_m,\ x_{m+1},\ x_m, \ldots,\ x_3,\ x_2,\ x_1 = 1\},$$
$$U = \{u_1 = 1,\ u_2,\ u_3, \ldots,\ u_m,\ u_{m+1},\ u_m, \ldots,\ u_3,\ u_2,\ -1\},$$
$$Y = \{y_1,\ y_2, \ldots,\ y_m,\ -y_m, \ldots,\ -y_2,\ -y_1\},$$
$$V = \{v_1,\ v_2, \ldots,\ v_m,\ -v_m, , \ldots,\ -v_2,\ -v_1\},$$

which have zero non periodic auto correlation function and $8m + 2$ the sum of two squares will be called *Turyn sequences* of lengths $n+1, n+1, n, n$.

Known Turyn sequences are given in [1] where the longer sequence is of length 2, 3, 4, 5, 6, 7, 8, 13, 15. They cannot exist for 11, 12, 17 or 18 and a complete machine search showed they do not exist for lengths 9, 10, 14 or 16. So the first unresolved cases (and unresolved since 1976) are lengths 19, 20 and 21.

Length	Sum of Squares	Sequences
m	$2m+1$	
12	$25 = 5^2$	(i) {−+++−+−+++−++}, {++−+−−+−+++++) {−−+−++−−−+++}, {−−+−−−++−+++} (ii) {−+++++−+++−+ −}, {++−++−+−++++ −) {+−−−−−++++−+}, {+−++−−++−−−+}
13	$27 = 4^2 + 3^2 + 1^2 + 1^2$	(i) {−−−++−+−−++−++}, {−−−+++−++++−+−} {+−++−+−−−−++ +}, {+−+++−+++−++ +} (ii) {−−+−+−++−+−−++}, {+−−++−−−−+++++) {−−+−+−−+++++ −), {+−+++−+++−++ +) (iii) {−++−++++−−−−+−}, {++++−+−−−++−+−} {−−+++−−++−++ −}, {+++++−+++−+− +} (iv) {−−+−−+++−−+−++}, {+−++++−−−+−−++) {−−+−+−−+++++ −}{+++++−+++−+− +}

Table 1: Base sequences of lengths $m+1, m+1, m, m$

Length	Sum of Squares	Sequences
m	$2m+1$	
13	$27 = 3^2 + 3^2 + 3^2$	(i) {− − − − + + − + + + + − + −}, {+ + + − + + − + + + + + − −} {+ + − − − + + + − + + − +}, {+ − − + + − + − + − + + +} (ii) {+ − + + − − − − + + + − − +}, {− + − + + − + + + − + + + +} {− + − − + + + + + + + − −}, {+ − − + + − + − + − + + +} (iii) {− − + + − + − − + − + − + +}, {+ − − − + + + + + + − + + +} {+ + + + − + + − − − − + +}, {+ − − + + − + + + − + − +} (iv) {− − + − − + + − − − + + + +}, {+ + + + + − + − + + − + − +} {+ + + − − + + − − + − + +}, {+ − + + + + + − + − − − +} (v) {− + − − + − + + + − − − + +}, {+ − + + + + + + − − + + − +} {− + − − + + + − + + + + −}, {+ − − − + − + + + + + − +} (vi) {− − − + − − + + + − + − + +}, {+ − + + − + + + − − + + + +} {− + + − + + − + + + + − −}, {+ − + − + − − − + + + + +}
15	$31 = 5^2 + 2^2 + 1^2 + 1^2$	see [1]
15	$31 = 3^2 + 3^2 + 3^2 + 2^2$	(i) {− − − + − + + − + − + + − − + +} {+ − + − − + − − + + + + + + + +} {+ + − − − − + + + − + + − − +} {+ − + − + − + + + − − + + + +} (ii) {− − − + − + + − + − + + − − + +} {+ − + − − + − − + + + + + + + +} {+ + − + − − + + + − + − − − +} {+ − + + + − + + + − − − + + +}
16	$33 = 5^2 + 2^2 + 2^2$	(i) {+ − + + + − − + − + + + − + + + −} {+ + + − + + − + + + + − − − + − +} {+ − − − + + + + + − + − − + − −} {+ − + + − + + + + − + + + − − −} (ii) {+ − − + − + + + − + + + − − + + +} {+ + − − + + + − + − + + + + + − −} {− − + − + − − + − − + + + + + −} {+ − + − + − − + − − + + + + + +}
16	$33 = 4^2 + 4^2 + 1^2$	sec [1], or (i) {− + + + + − − − − + − + − + + + −} {− − − + − − + + + − + + + + − − +} {− + + − + − + − − + + − + + − −} {+ + + + + − + + + + + − − + − +}
16	$33 = 4^2 + 3^2 + 2^2 + 2^2$	(i) {− − − − + + − + + + − + + + − + −} {− − + + + + − + + + − + + − + + +} {+ + − + − − − − + + − − + − + +} {− + + + + − + − + − − + + + + −}

Table 1(cont.): Base sequences of lengths $m+1, m+1, m, m$

Length	Sum of Squares	Sequences
m	$2m+1$	
		(ii) {− + − + + + − + + + − + + − − − −} {− − + + + + − + + + − + + − + + +} {+ + − − − − − + + − + − + + − +} {+ + + − + − + − − + + + + − − +} (iii) {− − + + − − + + − + − + − + − + +} {+ − − + + − + + − + − + + + + + +} {− + + − − − − + + + + − + + − −} {+ − + − − − − + + + + − + + + +}
17	$35 = 5^2 + 3^2 + 1^2$	**see [1],[7], or** (i) {− + + − + + − − − + + − + − + + + −} {+ − + + + + + − − + + + + − − + − −} {+ + + + + − − − + + − + + − + − +} {+ − + − + − − − + + − + + + + + +} (ii) {+ + + − + − + + − − + − + + − − − +} {− − + + + − − + − − + + + + + − + +} {+ + + + + − − − + + − + + − + − +} {+ − + − + − − − + + − + + + + + +}
17	$35 = 4^2 + 3^2 + 3^2 + 1^2$	(i) {− − − + + + + − + + − − + − + + + −} {− − + + + + − + − − + + + − + − + +} {+ + + + − − + + − − − + − − + − +} {+ − + − + − − + + − + + + + + + +}
18	$37 = 6^2 + 1^2$	(i) {+ + + + − − − − + − + − + − + − − + +} {+ − + − − + − − + + + − + + + + − − −} {+ − − + + + − + + − + + + + + − + −} {+ + + + + − + + − + + − − + + + − −}
18	$37 = 5^2 + 2^2 + 2^2 + 2^2$	(i) {− − + − + + − − − + − + − + + + + + +} {+ + + − − + + − + − + + + + − + + − +} {− + + + + − − − + − + − + + − − + −} {+ + − − + − − + + − − − + + + + + +}
18	$37 = 4^2 + 4^2 + 2^2 + 1^2$	(i) {+ + + + + − + + − − − − − + + − + − −} {+ − + − − − + − + − + + − + − + + + +} {− + + − + − − + + + − − + + − − + −} {+ + + + + − − − + + + − + + + − + +}
25	$51 = 7^2 + 1^2 + 1^2$	(i) {+ + + + + + + + − − + + + − − + − − + + − + − + − +} {− + + + + + + + − − + + + − − + − − + + − + − + − +} {+ + + + + + − − − − + + − − + + − + − − + − + − −} {+ − + − + − − + − + + − − + + − − − − + + + + + −}

Table 1(cont.): Base sequences of lengths $m+1, m+1, m, m$

2. Constructions and decompositions into squares

The results that we now give arise from a remarkable composition theorem due to C.H. Yang [13] which has been announced.

Let X, Y, U, W be *Turyn sequences* of lengths $n+1, n+1, n, n$. The sequences defined by

$$F = X/U,\ G = Y/0,\ H = W/0,\ E = (1) \qquad (1)$$

will be called *Yang nice sequences* (or *Turyn normal sequences*). Otherwise we use the notation and definitions of [1].

Theorem 1. *(Based on a Theorem of C.H. Yang): Let F, G, H, E be Yang nice sequences of length $4n+3$ constructed from Turyn sequences as in (1), and A, B, C, D be base sequences of lengths $m+p, m+p, m, m, (t = 2m+p)$. Then the sequences Q, R, S, T, given below, become 4-complementary sequences (that is the sum of the non-periodic auto-correlation functions is 0) of lengths $(4n+3)t$.*

$$\begin{aligned} Q &= (-B'e + Af' + Cg + Dh) \\ R &= (A'e + Bf' + Dg' - Ch') \\ S &= (-D'e - Cf + Ag' - Bh) \\ T &= (C'e - Df + Bg + Ah') \end{aligned}$$

Furthermore if we define sequences

$$P_1 = \frac{1}{2}(Q+R),\ P_2 = \frac{1}{2}(Q-R),\ P_3 = \frac{1}{2}(S+T),\ P_4 \frac{1}{2}(S-T),$$

then these sequences become T-sequences of length $(4n+3)t, t = 2m+p$.

Theorem 2. *Under the conditions stated in Theorem 1 the following sequences can be used, as in Theorem 1, to form disjoint T-sequences of lengths $(4n+3)(2m+p)$.*

$$\begin{aligned} Q &= (A'e + Bf' + Cg' + Dh') \\ R &= (-B'e + Af' + Dg - Ch) \\ S &= (-C'e - Df + Ag' + Bh) \\ T &= (-D'e - Cf - Bg + Ah') \end{aligned}$$

Turyn sequences of lengths $n+1, n+1, n, n$, exist for $n \le 7$, $n = 12$ or $n = 14$, hence we can construct Yang nice sequences for lengths $(4n+3)$ and so, from Theorems 1 and 2, 4-disjoint T-sequences of lengths $(4n+3)t$ where $t = 2m+p$ is the length of base sequences can be constructed.

Theorem 3. *Let a, b, c, d be the row sums of four suitable sequences, A, B, C, D of length $m+1, m+1, m, m$, so that $2m+1 = a^2+b^2+c^2+d^2$. Let f, g, h, and i, be the row sums of four Turyn sequences of length $4n+3$ so that $4n+3 = f^2+g^2+h^2+i^2$. Then using Yang's method to multiply by $4n+3$,* *as in Theorem 1, we get four disjoint T-sequences of lengths $(4n+3)(2m+1)$ corresponding to one of the decompositions:*

$$
\begin{aligned}
(4n+3)(2m+1) &= (af-bg+ch+di)^2+(ag+bf-ci+dh)^2 \\
&\quad +(ah-bi-cf-dg)^2+(ai+bh+cg-df)^2 \\
(4n+3)(2m+1) &= (bf-ag+ch+di)^2+(bg+af-ci+dh)^2 \\
&\quad +(bh-ai-cf-dg)^2+(bi+ah+cg-df)^2 \\
(4n+3)(2m+1) &= (af-bg+dh+ci)^2+(ag+bf-di+ch)^2 \\
&\quad +(ah-bi-df-cg)^2+(ai+bh+dg-cf)^2 \\
(4n+3)(2m+1) &= (bf-ag+dh+ci)^2+(bg+af-di+ch)^2 \\
&\quad +(bh-ai-df-cg)^2+(bi+ah+dg-cf)^2
\end{aligned}
$$

Corollary 4. *Let a, b, c, d be the row sums of suitable sequences of length $m+1$, $m+1, m, m$, so that $2m+1 = a^2+b^2+c^2+d^2$. Then using Yang's method to multiply by $4n+3$, as in Theorem 1, we get four disjoint T-sequences of lengths $(4n+3)(2m+1)$ corresponding to one of four decompositions indicated in Table 2.*

Theorem 5. *Let a, b, c, d be the row sums of four suitable sequences, A, B, C, D of length $m+1, m+1, m, m$, so that $2m+1 = a^2+b^2+c^2+d^2$. Let f, g, h, and i, be the row sums of four Turyn sequences of length $4n+3$ so that $4n+3 = f^2+g^2+h^2+i^2$. Then using Yang's method to multiply by $4n+3$, as in theorem 2, we get four disjoint T-sequences of lengths $(4n+3)(2m+1)$ corresponding to one of the decompositions:*

$$
\begin{aligned}
(4n+3)(2m+1) &= (ag+bf+ch+di)^2+(af-bg-ci+dh)^2 \\
&\quad +(ah+bi-cg-df)^2+(ai-bh+cf-dg)^2 \\
(4n+3)(2m+1) &= (bg+af+ch+di)^2+(bf-ag-ci+dh)^2 \\
&\quad +(bh+ai-cg-df)^2+(bi-ah+cf-dg)^2 \\
(4n+3)(2m+1) &= (ag+bf+dh+ci)^2+(af-bg-di+ch)^2 \\
&\quad +(ah+bi-dg-cf)^2+(ai-bh+df-cg)^2 \\
(4n+3)(2m+1) &= (bg+af+dh+ci)^2+(bf-ag-di+ch)^2 \\
&\quad +(bh+ai-dg-cf)^2+(bi-ah+df-cg)^2
\end{aligned}
$$

Corollary 6. *Let a, b, c, d be the row sums of suitable sequences of length $m+1$, $m+1, m, m$, so that $2m+1 = a^2+b^2+c^2+d^2$. Then using Yang's method to multiply by $4n+3$, as in Theorem 2, we get four disjoint T-sequences of lengths*

$(4n+3)(2m+1)$ *corresponding to one of four decompositions indicated in Table 3.*

Example: There are suitable sequences for $m = 5$ corresponding to $a = 1, b = 1$, $c = 3$ and $d = 0$. These give sequences of lengths 121 corresponding to:

$$121 = 8^2 + 7^2 + 2^2 + 2^2 \text{ giving excess } \sigma(H^1_{484}) = 9196.$$

$$121 = 8^2 + 5^2 + 4^2 + 4^2 \text{ giving excess } \sigma(H^2_{484}) = 10164.$$

They also give sequences of lengths 165 corresponding to:

$$165 = 8^2 + 8^2 + 6^2 + 1^2 \text{ giving excess } \sigma(H_{660}) = 15180.$$

$$165 = 10^2 + 6^2 + 5^2 + 2^2 \text{ giving excess } \sigma(H_{660}) = 15180.$$

These are examples of the method: the new sequences that arise from the decompositions and which lead to Hadamard matrices with maximum known excess are given in Table 2 and 3 are listed in Tables 6 and 7. Full details are given in a Technical Report (University College, ADFA, TR CS88/18) available from the authors.

Remark: We note that it is only the numbers $4n+3$ which are the sum of four (and not fewer) squares which actually lead to an inequivalent decomposition between Yang's construction and our modified construction (Theorem 2). However our modification does prove to be important for obtaining maximum known excess.

We have exhibited the new T-sequences in Tables 4 and 5 as they are possibly inequivalent to previously known sequences.

t		Sum of Squares	Comment	KF Upper Bound	Excess
165	15×11	$8^2 + 7^2 + 6^2 + 4^2$	$y = 15, m = 5$	16936	16500
287	7×41	$11^2 + 9^2 + 9^2 + 2^2$	$y = 7, m = 20$	38888	35588†
299	23×13	$17^2 + 3^2 + 1^2$	$y = 23, m = 6$	41328	40664†

Table 7: T-disjoint T-sequences giving Hadamard matrices with maximum known excess

† new maximum known excess

3. New excess results

We now use the new T-sequences, with the results of the previous section, as in Hammer, Levingston and Seberry [3] or Jenkins *et. al.* [4] to obtain Hadamard matrices of order $4t$ with maximum known excess.

These are given in two tables: Table 6 corresponds to Yang's construction and Table 7 to our new construction. We have more complete results for this section in the Technical report available from the authors referred to above.

4. Summary

The sequences constructed here can be combined with those of [1] and used to obtain orthogonal designs $OD(4t; t, t, t, t)$ for $t = 1, 3, \ldots, 41, 45, \ldots, 65, 67, 69, 75, 77, 81, 85, 87, 91, 93, 95, 99, 101, 105, 111, 115, \ldots, 125, 129, 133, 135, 141, \ldots, 147, 153, 155, 159, 161, 165, 169, 171, 175, 177, 183, 185, 187, 189, 195, 201, 203, 205, 207, 209, 217, 221, 225, 231, 235, 243, 245, 247, 253, 255, 259, 261, 265, 273, 275, 279, 285, 287, 289, 295, 297, 299$.

Williamson matrices are used with these orthogonal designs to form Hadamard matrices with the maximum known excess for order $4t$, $t = 141, 143, 153, 155, 165, 171, 175, 177, 185, 189, 195, 203, 205, 217, 221, 231, 235, 245, 247, 253, 255, 259, 261, 265, 273, 275, 279, 285, 287, 289, 295, 297, 299$.

4n+3		Decompositions
7	7(2m+1)	$(a-b+2c+d)^2+(a+b-c+2d)^2+(2a-b-c-d)^2+(a+2b+c-d)^2$ $(b-a+2c+d)^2+(b+a-c+2d)^2+(2b-a-c-d)^2+(b+2a+c-d)^2$ $(a-b+2d+c)^2+(a+b-d+2c)^2+(2a-b-d-c)^2+(a+2b+d-c)^2$ $(b-a+2d+c)^2+(b+a-d+2c)^2+(2b-a-d-c)^2+(b+2a+d-c)^2$
11	11(2m+1)	$(3a-b+c)^2+(a+3b+d)^2+(a-3c-d)^2+(b+c-3d)^2$ $(3b-a+c)^2+(b+3a+d)^2+(b-3c-d)^2+(a+c-3d)^2$ $(3a-b+d)^2+(a+3b+c)^2+(a-3d-c)^2+(b+d-3c)^2$ $(3b-a+d)^2+(b+3a+c)^2+(b-3d-c)^2+(a+d-3c)^2$
15	15(2m+1)	$(3a-b+2c+d)^2+(a+3b-c+2d)^2+(2a-b-3c-d)^2+(a+2b+c-3d)^2$ $(3b-a+2c+d)^2+(b+3a-c+2d)^2+(2b-a-3c-d)^2+(b+2a+c-3d)^2$ $(3a-b+2d+c)^2+(a+3b-d+2c)^2+(2a-b-3d-c)^2+(a+2b+d-3c)^2$ $(3b-a+2d+c)^2+(b+3a-d+2c)^2+(2b-a-3d-c)^2+(b+2a+d-3c)^2$
19	19(2m + 1)	$(3a-b+3c)^2+(a+3b+3d)^2+(3a-3c-d)^2+(3b+c-3d)^2$ $(3b-a+3c)^2+(b+3a+3d)^2+(3b-3c-d)^2+(3a+c-3d)^2$ $(3a-b+3d)^2+(a+3b+3c)^2+(3a-3d-c)^2+(3b+d-3c)^2$ $(3b-a+3d)^2+(b+3a+3c)^2+(3b-3d-c)^2+(3a+d-3c)^2$
23	23(2m + 1)	$(3a-b+2c+3d)^2+(a+3b-3c+2d)^2+(2a-3b-3c-d)^2+(3a+2b+c-3d)^2$ $(3b-a+2c+3d)^2+(b+3a-3c+2d)^2+(2b-3a-3c-d)^2+(3b+2a+c-3d)^2$ $(3a-b+2d+3c)^2+(a+3b-3d+2c)^2+(2a-3b-3d-c)^2+(3a+2b+d-3c)^2$ $(3b-a+2d+3c)^2+(b+3a-3d+2c)^2+(2b-3a-3d-c)^2+(3b+2a+d-3c)^2$
27	27(2m + 1)	$(5a-b+c)^2+(a+5b+d)^2+(a-5c-d)^2+(b+c-5d)^2$ $(5b-a+c)^2+(b+5a+d)^2+(b-5c-d)^2+(a+c-5d)^2$ $(5a-b+d)^2+(a+5b+c)^2+(a-5d-c)^2+(b+d-5c)^2$ $(5b-a+d)^2+(b+5a+c)^2+(b-5d-c)^2+(a+d-5c)^2$
31	31(2m + 1)	$(5a-b+2c+d)^2+(a+5b-c+2d)^2+(2a-b-5c-d)^2+(a+2b+c-5d)^2$ $(5b-a+2c+d)^2+(b+5a-c+2d)^2+(2b-a-5c-d)^2+(b+2a+c-5d)^2$ $(5a-b+2d+c)^2+(a+5b-d+2c)^2+(2a-b-5d-c)^2+(a+2b+d-5c)^2$ $(5b-a+2d+c)^2+(b+5a-d+2c)^2+(2b-a-5d-c)^2+(b+2a+d-5c)^2$
51	51(2m + 1)	$(7a-b+c)^2+(a+7b+d)^2+(a-7c-d)^2+(b+c-7d)^2$ $(7b-a+c)^2+(b+7a+d)^2+(b-7c-d)^2+(a+c-7d)^2$ $(7a-b+d)^2+(a+7b+c)^2+(a-7d-c)^2+(b+d-7c)^2$ $(7b-a+d)^2+(b+7a+c)^2+(b-7d-c)^2+(a+d-7c)^2$

Table 2: Decomposition arising from Yang's composition $(4n+3)(2m+1)$ where $2m+1 = a^2+b^2+c^2+d^2$ using Theorem 1.

4n+3		Decompositions
59	59(2m + 1)	$(7a - b + 3c)^2 + (a + 7b + 3d)^2 + (3a - 7c - d)^2 + (3b + c - 7d)^2$ $(7b - a + 3c)^2 + (b + 7a + 3d)^2 + (3b - 7c - d)^2 + (3a + c - 7d)^2$ $(7a - b + 3d)^2 + (a + 7b + 3c)^2 + (3a - 7d - c)^2 + (3b + d - 7c)^2$ $(7b - a + 3d)^2 + (b + 7a + 3c)^2 + (3b - 7d - c)^2 + (3a + d - 7c)^2$

Table 2 (cont'd)

4n+3		Decompositions
7	7(2m+1)	$(a + b + 2c + d)^2 + (a - b - c + 2d)^2 + (2a + b - c - d)^2 + (a - 2b + c - d)^2$ $(b + a + 2c + d)^2 + (b - a - c + 2d)^2 + (2b + a - c - d)^2 + (b - 2a + c - d)^2$ $(a + b + 2d + c)^2 + (a - b - d + 2c)^2 + (2a + b - d - c)^2 + (a - 2b + d - c)^2$ $(b + a + 2d + c)^2 + (b - a - d + 2c)^2 + (2b + a - d - c)^2 + (b - 2a + d - c)^2$
11	11(2m+1)	$(a + 3b + c)^2 + (3a - b + d)^2 + (a - c - 3d)^2 + (-b + 3c - d)^2$ $(b + 3a + c)^2 + (3b - a + d)^2 + (b - c - 3d)^2 + (-a + 3c - d)^2$ $(a + 3b + d)^2 + (3a - b + c)^2 + (a - d - 3c)^2 + (-b + 3d - c)^2$ $(b + 3a + d)^2 + (3b - a + c)^2 + (b - d - 3c)^2 + (-a + 3d - c)^2$
15	15(2m+1)	$(a + 3b + 2c + d)^2 + (3a - b - c + 2d)^2 + (2a + b - c - 3d)^2 + (a - 2b + 3c - d)^2$ $(b + 3a + 2c + d)^2 + (3b - a - c + 2d)^2 + (2b + a - c - 3d)^2 + (b - 2a + 3c - d)^2$ $(a + 3b + 2d + c)^2 + (3a - b - d + 2c)^2 + (2a + b - d - 3c)^2 + (a - 2b + 3d - c)^2$ $(b + 3a + 2d + c)^2 + (3b - a - d + 2c)^2 + (2b + a - d - 3c)^2 + (b - 2a + 3d - c)^2$
19	19(2m + 1)	$(a + 3b + 3c)^2 + (3a - b + 3d)^2 + (3a - c - 3d)^2 + (-3b + 3c - d)^2$ $(b + 3a + 3c)^2 + (3b - a + 3d)^2 + (3b - c - 3d)^2 + (-3a + 3c - d)^2$ $(a + 3b + 3d)^2 + (3a - b + 3c)^2 + (3a - d - 3c)^2 + (-3b + 3d - c)^2$ $(b + 3a + 3d)^2 + (3b - a + 3c)^2 + (3b - d - 3c)^2 + (-3a + 3d - c)^2$
23	23(2m + 1)	$(a + 3b + 2c + 3d)^2 + (3a - b - 3c + 2d)^2 + (2a + 3b - c - 3d)^2 + (3a - 2b + 3c - d)^2$ $(b + 3a + 2c + 3d)^2 + (3b - a - 3c + 2d)^2 + (2b + 3a - c - 3d)^2 + (3b - 2a + 3c - d)^2$ $(a + 3b + 2d + 3c)^2 + (3a - b - 3d + 2c)^2 + (2a + 3b - d - 3c)^2 + (3a - 2b + 3d - c)^2$ $(b + 3a + 2d + 3c)^2 + (3b - a - 3d + 2c)^2 + (2b + 3a - d - 3c)^2 + (3b - 2a + 3d - c)^2$
27	27(2m + 1)	$(a + 5b + c)^2 + (5a - b + d)^2 + (a - c - 5d)^2 + (-b + 5c - d)^2$ $(b + 5a + c)^2 + (5b - a + d)^2 + (b - c - 5d)^2 + (-a + 5c - d)^2$ $(a + 5b + d)^2 + (5a - b + c)^2 + (a - d - 5c)^2 + (-b + 5d - c)^2$ $(b + 5a + d)^2 + (5b - a + c)^2 + (b - d - 5c)^2 + (-a + 5d - c)^2$
31	31(2m + 1)	$(a + 5b + 2c + d)^2 + (5a - b - c + 2d)^2 + (2a + b - c - 5d)^2 + (a - 2b + 5c - d)^2$ $(b + 5a + 2c + d)^2 + (5b - a - c + 2d)^2 + (2b + a - c - 5d)^2 + (b - 2a + 5c - d)^2$ $(a + 5b + 2d + c)^2 + (5a - b - d + 2c)^2 + (2a + b - d - 5c)^2 + (a - 2b + 5d - c)^2$ $(b + 5a + 2d + c)^2 + (5b - a - d + 2c)^2 + (2b + a - d - 5c)^2 + (b - 2a + 5d - c)^2$
51	51(2m + 1)	$(a + 7b + c)^2 + (7a - b + d)^2 + (a - c - 7d)^2 + (-b + 7c - d)^2$ $(b + 7a + c)^2 + (7b - a + d)^2 + (b - c - 7d)^2 + (-a + 7c - d)^2$ $(a + 7b + d)^2 + (7a - b + c)^2 + (a - d - 7c)^2 + (-b + 7d - c)^2$ $(b + 7a + d)^2 + (7b - a + c)^2 + (b - d - 7c)^2 + (-a + 7d - c)^2$
59	59(2m + 1)	$(a + 7b + 3c)^2 + (7a - b + 3d)^2 + (3a - c - 7d)^2 + (-3b + 7c - d)^2$ $(b + 7a + 3c)^2 + (7b - a + 3d)^2 + (3b - c - 7d)^2 + (-3a + 7c - d)^2$ $(a + 7b + 3d)^2 + (7a - b + 3c)^2 + (3a - d - 7c)^2 + (-3b + 7d - c)^2$ $(b + 7a + 3d)^2 + (7b - a + 3c)^2 + (3b - d - 7c)^2 + (-3a + 7d - c)^2$

Table 3: Decomposition arising from Yang's composition $(4n+3)(2m+1)$ where $2m+1 = a^2 + b^2 + c^2 + d^2$ using Theorem 2.

Length	Sum of Squares		Sequences
31	$5^2 + 2^2 + 1^2 + 1^2$	see [1]	
31	$3^2 + 3^2 + 3^2 + 2^2$	(i)	{ 1 0 0 -1 0 -1 1 1 1 -1 0 1 0 0 1 0_{16} } { 0_{16} -1 0 0 -1 1 0 -1 1 0 1 1 0 0 1 1 } { 0_{15} 1 0 1 -1 0 0 -1 0 0 1 0 0 1 1 0 0 } { 0 -1 1 0 1 0_5 -1 0 1 1 0_{17} }
		(ii)	{ 1 0 0 1 0 -1 1 1 1 -1 0 -1 0 0 1 0_{16} } { 0_{16} -1 0 0 -1 1 0 -1 1 0 1 1 0 0 1 1 } { 0_{15} 1 0 1 -1 0 0 -1 0 0 1 0 0 1 1 0 0 } { 0 -1 1 0 1 0_5 -1 0 1 1 0_{17} }
33	$5^2 + 2^2 + 2^2$	(i)	{ 0_{16} 1 0 1 0 1 0 -1 1 0 1 1 0 -1 0 1 0 0 } { 1 -1 0_3 1 1 1 1 -1 1 0_3 -1 -1 0_{17} } { 0 0 1 1 -1 0_6 1 1 -1 0_{19} } { 0_{17} -1 0 1 0 -1 0 0 -1 0 0 1 0 1 0 1 -1 }
		(ii)	{ 0_{16} 1 0 -1 0 0 1 1 0_3 1 1 0 0 1 0 0 } { 0 -1 1 -1 1 -1 -1 1 -1 -1 1 1 1 1 1 1 0_{18} } { 1 0_{14} 1 0_{17} } { 0_{17} -1 0 1 0 -1 0 0 1 -1 1 0 0 -1 -1 0 1 1 }
33	$4^2 + 4^2 + 1^2$	see [1] or	
		(i)	{ 0 1 1 0 1 -1 1 0 0 1 1 -1 0 1 -1 0_{18} } { 1 0 0 1 0_3 1 1 0_3 -1 0 0 1 0_{17} } { 0_{16} -1 0 0 1 0 -1 0_5 1 0 1 0_3 } { 0_{17} 1 1 0 1 0 -1 -1 -1 1 -1 0 -1 0 1 1 -1 }
33	$4^2 + 3^2 + 2^2 + 2^2$	(i)	{ 0_{16} -1 -1 0 0 1 1 -1 1 1 1 -1 1 1 0 0 1 0 } { 0_{18} 1 1 0_9 -1 1 0 1 } { 0 1 0 1 0 -1 0 -1 1 0 -1 0 1 0 1 0_{18} } { -1 0 1 0 1 0 1 0 0 -1 0 1 0 1 0 -1 0_{17} }
		(ii)	{ 0_{16} -1 0 0 1 1 1 -1 1 1 1 -1 1 1 -1 0_3 } { 0_{17} -1 1 0_{11} 1 1 1 } { 1 1 0 -1 0 -1 0_4 1 0 1 0 -1 1 0_{17} } { 0 0 1 0 1 0 1 -1 -1 1 0 1 0 -1 0_{19} }
		(iii)	{ 0_{17} -1 0 1 0 -1 1 1 -1 1 -1 1 0 1 0 1 1 } { 0_{16} 1 0 -1 0 1 0_7 1 0 1 0 0 } { 0 0 1 -1 -1 -1 -1 1 1 1 1 -1 1 1 0_{19} } { 1 -1 0_{12} 1 1 0_{17} }

Table 4: new T-sequences of length t

Length	Sum of Squares	Sequences
35	$5^2 + 3^2 + 1^2$	see [1, 7], or (i) { 1 0 1 0 1–1–1–1 1 1–1 1 1 0 1 0 1 0_{18}} { 0_{19} 1 0 1 1 0–1–1 1 1 0 1–1 0 1 0–1} { 0_{17} 1–1 0 1 0 0 1 0_4 1 0 0–1 0–1 0} { 0 1 0 1 0_9 –1 0–1 0_{19}} (ii) { 1 0 1 0 1–1–1–1 1 1–1 1 1 0 1 0 1 0_{18}} { 0_{19} 1 0 1–1 0 1–1–1 1 0 1 1 0–1 0 1} { 0_{17}–1–1 0 1 0 0–1 0_4 1 0 0 1 0 1 0} [0 1 0 1 0_9–1 0–1 0_{19}}
35	$4^2 + 3^2 + 3^2 + 1^2$	(i) [1 0 1 0 0–1 0 1 0–1 0 1 0 0 1 0 1 0_{18}} { 0–1 0–1 1 0–1 0 1 0 1 0 1 1 0 1 0_{19}} { 0_{17}–1–1 0 1 1 1 0_6 1–1 1 0 1 0} { 0_{19} 1 0_3–1 1–1–1 1 1 0_3–1 0 1)
37	$6^2 + 1^2$	(i) { 1 0 0 1 1 0 0 1 0 0 1 0 0 1 1 0 0–1 0_{19}} { 0_{18} 1 0 1 0–1 0–1–1 1 0 1–1 1 0 1 0–1 0 0) [0_{19} 1 0 1 0–1 0_3–1 0_3–1 0–1 0 1 1} { 0–1–1 0 0 1–1 0 1–1 0 1 1 0 0–1 1 0_{20}}
37	$5^2 + 2^2 + 2^2 + 2^2$	(i) { 0_{20} 1–1 0 1 0–1 0_3 1 0 1 0 1 1 0 1} { 0_{18} 1 1 0 0–1 0 1 0 1–1 1 0 1 0–1 0 0–1 0} { 0 1 0 0 1–1–1 0 1–1 0–1 1 1 0 0 1 0_{20}) { 1 0–1–1 0_3 1 0 0–1 0_3 1 1 0 1 0_{19}}
37	$4^2 + 4^2 + 2^2 + 1^2$	(i) { 0 1 1 0 1–1–1 0 1 1 0–1 1 1 0–1 1 0_{20}) { 1 0 0 1 0_3–1 0 0 1 0_3 1 0 0 1 0_{19}) { 0_{18} 1 0 1 0 0–1 1 0 0–1 0 0–1 1 0 0 1 0 0] { 0_{19}–1 0–1–1 0 0–1 1 0 1 1 0 0–1 1 0 1 1}

Table 4(cont.): new T-sequences of length t

4n+3	t	(4n+3)t Sum of Squares	Sequences
11	3	$33 = 5^2 + 2^2 + 2^2 + 0^2$	{1 0_5 −1 0 1 0 0 0 1 0 1 0 0 0 1 0 −1 0 0 0 1 0_5 1 0 0} {0 1 1 0_4 −1 0_5 1 0_5 1 0_5 1 −1 0_4 −1 0} {0_4 −1 −1 0 0 0 −1 0 −1 0 0 0 1 0 −1 0 0 0 1 0 1 0_4 1 −1 0 0 0} {0 0 0 −1 0_6 1 0_5 1 0_5 −1 0_4 1 0_4 −1}
11	5	$55 = 7^2 + 2^2 + 1^2 + 1^2$	{1 1 0_8 −1 −1 0 1 −1 0_5 1 1 0 1 −1 0_5 1 1 0 −1 1 0_5 1 1 0_8 1 0_4} {0 0 1 1 −1 0_7 −1 0_9 1 0_9 1 0_9 1 −1 1 0_7 −1 −1 0 0} {0_7 −1 −1 1 0_5 −1 −1 0 −1 1 0_5 1 1 0 −1 1 0_5 1 1 0 1 −1 0_7 1 −1 1 0_5} {0_5 −1 −1 0_{10} 1 0_9 1 0_9 −1 0_7 1 1 0_6 1 −1}
11	7	$77 = 6^2 + 4^2 + 4^2 + 3^2$	{1 1 −1 0 0 1 0_8 −1 −1 1 0 1 0 1 0_7 1 1 −1 0 1 0 1 0_7 1 1 −1 0 −1 0 −1 0_7 1 1 −1 0 0 −1 0_8 1 0_6} {0 0 0 1 1 0 1 0_{10} −1 0 −1 0_{11} 1 0 1 0_{11} 1 0 1 0_{11} 1 −1 0 −1 0_8 1 −1 −1 0 0 0} {0_{10} −1 −1 0 −1 0_7 −1 −1 1 0 −1 0 −1 0_7 1 1 −1 0 −1 0 −1 0_7 1 1 −1 0 1 0 1 0_{10} 1 −1 0 −1 0_5 1 0} {0_7 −1 −1 1 0 0 −1 0_{11} 1 0 −1 0_{11} 1 0 −1 0_{11} −1 0 1 0_8 1 1 −1 0 0 −1 0_5 −1 0 −1}
15	3	$45 = 6^2 + 3^2 + 0^2 + 0^2$	{−1 0 1 0_3 1 0_5 −1 0_5 1 0_5 1 0_5 1 0_5 1 0 1 0_3 1 0 0} {0 −1 0_5 1 1 0_4 −1 1 0_4 1 −1 0_4 1 −1 0_4 1 1 0_4 1 0_5 −1 0} {0_3 1 0 −1 0_4 1 −1 0_4 −1 −1 0_4 −1 −1 0_4 1 1 0_4 1 −1 0_3 1 0 1 0_3} {0_4 1 0_4 −1 0_5 −1 0_5 1 0_5 1 0_5 −1 0_6 1 0_3 −1}
15	5	$75 = 7^2 + 5^2 + 1^2 + 0^2$	{−1 −1 0 1 −1 0_5 1 1 0_8 −1 −1 0_8 1 1 0_8 1 1 0_8 1 1 0_8 1 1 0 1 −1 0_5 1 0_4} {0 0 −1 0_9 1 1 −1 0_7 −1 1 −1 0_7 1 −1 1 0_7 1 −1 1 0_7 1 1 −1 0_7 1 0_8 −1 −1 0 0} {0_5 1 1 0 −1 1 0_7 1 −1 1 0_7 −1 −1 1 0_7 −1 −1 1 0_7 1 1 −1 0_7 1 −1 1 0_5 1 1 0 1 −1 0_5} {0_7 1 0_7 −1 −1 0_8 −1 −1 0_8 1 1 0_8 1 1 0_8 −1 −1 0_{10} 1 0_5 1 −1}
19	3	$57 = 7^2 + 2^2 + 2^2 + 0^2$	{1 0_5 −1 0 1 0_3 1 0 1 0_3 −1 0 −1 0_3 −1 0 1 0_3 1 0 1 0_3 1 0 1 0_3 1 0 −1 0_3 1 0_5 1 0 0} {0 1 1 0_4 −1 0_5 1 0_5 −1 0_5 −1 0_5 1 0_5 1 0_5 1 0_5 1 −1 0_4 −1 0} {0_4 −1 −1 0_3 −1 0 −1 0_3 1 0 −1 0_3 1 0 −1 0_3 1 0 1 0_3 −1 0 1 0_3 1 0 −1 0_3 1 0 1 0_4 1 −1 0_3} {0_3 −1 0_6 1 0_5 1 0_5 −1 0_5 1 0_5 1 0_5 1 0_5 −1 0_4 1 0_4 −1}

Table 5: Possibly inequivalent T-sequences of length $(4n+3)t$

			$-1 -1\ 0\ 1 -1\ 0_5\ 1\ 1\ 0 -1\ 1\ 0_5\ 1\ 1\ 0\ 1 -1\ 0_7\ 1 -1\ 1\ 0_5\}$ $\{0_5 -1 -1\ 0_{10}\ 1\ 0_9\ 1\ 0_9 -1\ 0_9\ 1\ 0_9\ 1\ 0_9\ 1\ 0_9 -1\ 0_7\ 1\ 1\ 0_6\ 1 -1\}$
23	3	$69 = 6^2 + 5^2 + 2^2 + 2^2$	$\{-1\ 0\ 1\ 0_3\ 1\ 0_5 -1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0$ $1\ 0_3\ 1\ 0\ 0\}$ $\{0 -1\ 0_5\ 1\ 1\ 0_4 -1\ 1\ 0_4\ 1\ 1\ 0_4 -1 -1\ 0_4 -1 -1\ 0_4\ 1\ 1\ 0_4\ 1\ 1\ 0_4\ 1 -1$ $0_4\ 1\ 1\ 0_4\ 1\ 0_5 -1\ 0\}$ $\{0_3\ 1\ 0 -1 0_4\ 1 -1 0_4 -1 -1 0_4\ 1 -1 0_4\ 1 -1 0_4 -1\ 1 0_4 -1\ 1 0_4\ 1 -1 0_4\ 1\ 1$ $0_4\ 1 -1 0_3\ 1\ 0\ 1 0_3\}$ $\{0_4\ 1\ 0_4 -1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5 -1\ 0_6\ 1\ 0_3 -1\}$
27	3	$81 = 6^2 + 5^2 + 4^2 + 2^2$	$\{1\ 0_5 -1\ 0\ 1\ 0_3\ 1\ 0\ 1\ 0_3 -1\ 0\ 1\ 0_3\ 1\ 0 -1\ 0_3\ 1\ 0 -1\ 0_3 -1\ 0 -1\ 0_3$ $-1\ 0\ 1\ 0_3\ 1\ 0 -1 0_3\ 1\ 0 -1\ 0_3\ 1\ 0\ 1\ 0_3\ 1\ 0 -1\ 0_3\ 1\ 0_5\ 1\ 0\ 0\}$ $\{0\ 1\ 1\ 0_4 -1\ 0_5\ 1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0_5$ $1\ 0_5\ 1 -1\ 0_4 -1\ 0\}$ $\{0_4 -1 -1\ 0_3 -1\ 0 -1\ 0_3\ 1\ 0 -1\ 0_3 -1\ 0 -1\ 0_3 -1\ 0 -1\ 0_3\ 1\ 0\ 1\ 0_3 -1$ $0\ 1\ 0_3 -1\ 0 -1\ 0_3 -1\ 0 -1\ 0_3\ 1\ 0\ 1\ 0_3\ 1\ 0 -1\ 0_3\ 1\ 0\ 1\ 0_4\ 1 -1\ 0_3\}$ $\{0_3 -1\ 0_6\ 1\ 0_5\ 1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5 -1$ $0_4\ 1\ 0_4 -1\}$
31	3	$93 = 8^2 + 4^2 + 3^2 + 2^2$	$\{-1\ 0\ 1\ 0_3\ 1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0\ 0\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5$ $-1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0\ 1\ 0_3\ 1\ 0\ 0\}$ $\{0 -1\ 0_5\ 1\ 1\ 0_4 -1\ 1\ 0_4\ 1 -1\ 0_4\ 1\ 1\ 0_4\ 1 -1\ 0_4 -1\ 1\ 0_4 -1\ 1\ 0_4\ 1$ $-1\ 0_4\ 1 -1\ 0_4 -1 -1\ 0_3\ 0\ 1 -1\ 0_4\ 1 -1\ 0_4\ 1\ 1\ 0_4\ 1\ 0_5 -1\ 0\}$ $\{0_3\ 1\ 0 -1\ 0_4\ 1 -1\ 0_4 -1 -1\ 0_4 -1 -1\ 0_4 -1\ 1\ 0_3\ 0 -1 -1\ 0_4 -1 -1\ 0_4\ 1\ 1$ $0_4\ 1\ 1\ 0_4 -1 -1\ 0_4\ 1 -1\ 0_4 -1 -1\ 0_4\ 1\ 1\ 0_4\ 1 -1\ 0_3\ 1\ 0\ 1\ 0_3\}$ $\{0_4\ 1\ 0_4 -1\ 0_5 -1\ 0_5\ 1\ 0_5 -1\ 0_5\ 1\ 0_5 -1\ 0_5 -1\ 0_5\ 1\ 0_5\ 1\ 0_5\ 1\ 0\ 0$ $0_3\ 1\ 0_5\ 1\ 0_5 -1\ 0_6\ 1\ 0_5 -1\}$

Table 5(cont.): Possibly inequivalent T-sequences of length $(4n+3)t$

t		Sum of Squares	Comment	KF ** Upper Bound	Excess
141	3×47	$11^2 + 4^2 + 2^2$	y = 3, m = 23	13380	12408†
143	11×13	$7^2 + 7^2 + 6^2 + 3^2$	y = 11, m = 6	13676	13156†
153	9×17	$12^2 + 2^2 + 2^2 + 1^2$	y = 9, m = 8	15108	14688†
155	5×31	$12^2 + 3^2 + 1^2 + 1^2$	y = 5, m = 15	15404	14880†
165	11×15	$8^2 + 7^2 + 6^2 + 4^2$	y = 11, m = 7	16936	16500
171	9×19	$13^2 + 1^2 + 1^2$	y = 9, m = 9	17880	17784
171	19×9	$13^2 + 1^2 + 1^2$	y = 19, m = 4	17880	17784
171	19×9	$11^2 + 5^2 + 4^2 + 3^2$	y = 19, m = 4	17880	17784
175	5×35	$13^2 + 2^2 + 1^2 + 1^2$	y = 5, m = 17	18496	18200†
177	59×3	$9^2 + 8^2 + 4^2 + 4^2$	y = 59, m = 1	18808	17700
185	5×37	$13^2 + 4^2$	y = 5, m = 18	20092	19240†
189	21×9	$9^2 + 6^2 + 6^2 + 6^2$	y = 21, m = 4	20756	20412
195	15×13	$9^2 + 7^2 + 7^2 + 4^2$	y = 15, m = 6	21780	21060
203	7×29	$9^2 + 9^2 + 5^2 + 4^2$	y = 7, m = 14	23108	21924
205	5×41	$14^2 + 2^2 + 2^2 + 1^2$	y = 5, m = 20	23452	22960†
217	7×31	$9^2 + 8^2 + 6^2 + 6^2$	y = 7, m = 15	25544	25172
221	17×13	$10^2 + 7^2 + 6^2 + 6^2$	y = 17, m = 6	26264	25636
231	11×21	$15^2 + 2^2 + 1^2 + 1^2$	y = 11, m = 10	28064	27720
235	5×47	$9^2 + 9^2 + 8^2 + 3^2$	y = 5, m = 23	28784	27260†
245	7×35	$9^2 + 8^2 + 8^2 + 6^2$	y = 7, m = 17	30636	30380†
247	19×13	$9^2 + 9^2 + 7^2 + 6^2$	y = 19, m = 6	31020	30628
253	11×23	$10^2 + 9^2 + 6^2 + 6^2$	y = 11, m = 11	32180	31372†
253	23×11	$10^2 + 8^2 + 8^2 + 5^2$	y = 23, m = 5	32180	31372†
255	15×17	$11^2 + 7^2 + 7^2 + 6^2$	y = 15, m = 8	32572	31620†
255	15×17	$10^2 + 9^2 + 7^2 + 5^2$	y = 15, m = 8	32572	31620†
259	7×37	$11^2 + 8^2 + 7^2 + 5^2$	y = 7, m = 18	33332	32116†
259	7×37	$9^2 + 9^2 + 9^2 + 4^2$	y = 7, m = 18	33332	32116†
261	9×29	$10^2 + 10^2 + 6^2 + 5^2$	y = 9, m = 14	33708	32364†
261	9×29	$10^2 + 9^2 + 8^2 + 4^2$	y = 9, m = 14	33708	32364†
265	53×5	$12^2 + 7^2 + 6^2 + 6^2$	y = 53, m = 2	34476	32860†
273	13×21	$16^2 + 3^2 + 2^2 + 2^2$	y = 13, m = 10	36036	34944†
275	11×25	$9^2 + 9^2 + 8^2 + 7^2$	y = 11, m = 12	36432	36300†
279	31×9	$13^2 + 7^2 + 6^2 + 5^2$	y = 31, m = 4	37240	34596†
279	31×9	$11^2 + 10^2 + 7^2 + 3^2$	y = 31, m = 4	37240	34596†
285	15×19	$11^2 + 8^2 + 8^2 + 6^2$	y = 15, m = 9	38472	37620†
285	15×19	$10^2 + 10^2 + 7^2 + 6^2$	y = 15, m = 9	38472	37620†
287	7×41	$11^2 + 11^2 + 6^2 + 3^2$	y = 7, m = 20	38888	35588†
289	17×17	17^2	y = 17, m = 8	39304	39304*†
295	5×59	$17^2 + 2^2 + 1^2 + 1^2$	y = 5, m = 29	40512	40120†
297	11×27	$17^2 + 2^2 + 2^2$	y = 11, m = 13	40920	40392†
297	33×9	$17^2 + 2^2 + 2^2$	y = 33, m = 4	40920	40392†

Table 6: 4-disjoint T-sequences giving Hadamard matrices with maximum known excess

* means the matrix constructed has maximal excess

† new maximum known excess

** see Kounias and Farmakis [6]

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